PAS-003-1162003

Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

August / September - 2020

Mathematics

Topology - II : CMT - 2003

Faculty Code : 003
Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer any **seven** of the following : **7x2=14**

(1) Define : Hausdorff space and Normal space.

(2) Show that (X, T) is T_1 if the cofinite topology is weaker than T.

(3) Prove that, homeomorphic image of a $T_{3\sim}$ space is T_3.

(4) State : Tietze's extension theorem.

(5) Define : Completely regular space. Also show that a completely regular space is regular.

(6) Is $\mathbb{N}$ compact with cofinite topology ? Justify your answer.

(7) Define finite intersection property with an example.

(8) Define : Uniform continuous function with an example.

(9) Show that every convergent sequence is Cauchy sequence.

(10) Prove that $\mathbb{R}$ with standard topology is regular.

2 Attempt any **two** : **2x7=14**

(1) Prove that X is T if and only if every single subset of X is closed.

(2) Prove that every compact subset of a T_2 space is closed.

(3) Show that a metric space is complete if every Cauchy sequence in X has a convergent subsequence.

3 Attempt any one : **1x14=14**

(1) (i) Define : Regular space. Prove that a subspace of a regular space is regular.

(ii) Prove that a T_1-space is normal if and only if given a closed set A and an open set U containing A, there exists an open set V such that $A \subseteq V \subseteq \overline{V} \subseteq U$.

(2) Every regular space with a countable basis is normal.

4 Answer any **two** of the following : **2x7=14**

(1) State and prove Tube Lemma.

(2) State and prove Lebesgue's Covering Lemma.

(3) Prove that X is compact if and only if whenever C is a collection of closed sets having finite intersection property, the intersection of all elements of C is non-empty.

5 Attempt any **two** : **2x7=14**

(1) Let X be a T_1-space and $A \subseteq X$. The prove that the point x is a limit point of A if and only if every neighbourhood of x contains infinitely many points of A.

(2) Prove that every regular space is Hausdoff. Is the converse true ? Justify.

(3) Prove that every compact space is limit point compact.

(4) A subspace of a complete metric space is complete if and only if it is closed.
